

# Discounting and Habit Formation of Exhaustible Resource Use

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# Motivation

- Most energy-environmental policy models are based on the Ramsey=Cass=Koopmans (simply, Ramsey) model.
  - a basis in economic growth theory
  - Example: Nordhaus' versions of the DICE model
- In the Ramsey model framework, the welfare is defined as:

- discrete time

$$W = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} U(c_t)$$

- continuous time

$$W \equiv \int_0^{\infty} U(c_t) e^{-\rho t} dt$$

This is called the discounted utility (DU) form, first proposed by Samuelson (1937, RES).

- When uncertain factors are considered, it is extended to the concept of expected utility.

$$W_t \equiv E_t \left[ \int_t^{\infty} U(c_s) e^{-\rho(s-t)} ds \right]$$

# Motivation, cont'd

- The DU form (inc., its expected utility version) is one of the most standard formulations in economic theory.
  - It is a conceptual device.
  - However, it materializes many important conceptions in economics, in particular time preference, uncertain future and risks in economic valuation.
  - The constant  $\rho$  is called *social time preference*.

$$W \equiv \int_0^{\infty} U(c_t) e^{-\rho t} dt$$

- The pros and cons about energy-environmental policy modeling and its resulting recommendations are mostly associated with the DU form, in particular the concept of social time preference  $\rho$ .
  - In fact, some recent studies in economics criticize the DU form and suggest alternative forms.

# Climate change policy debate

- The effects of time preference and discounting factors on the properties of the economic dynamics have been a central issue in economic policy debates.
  - Recently, the study report by Stern (2007), known as the “Stern Review,” suggested that a prompt action for climate change is needed.
  - The report created strong pros and cons in not only the policy arena but also the academia. One of the most debatable issues was the treatment of discount factors:
    - Nordhaus (2007), for example, criticized the Review, stating that the Review was assuming very low discount rates, and that the setting helps to explain most parts of its unusual conclusions.

# Studies on Time Preference

- In welfare economics
  - Specifying discount factors falls in moral philosophy (e.g., Arrow and Kurz, 1970).
  - Recent studies on this line include those on hyperbolic and/or Gamma discounting (e.g., Weitzman, 2001: *AER*).
- In macroeconomic theory
  - Many studies have been done on models in which time preference depends on endogenous economic variables.
  - Among those, habit formation models are popular in these days. “Habit formation” means that a history of consumption determines time preference. (e.g., Obstfeld, 1990: *J. Monetary Econ.*)
  - A classical one in this category includes the Uzawa-Epstein time preference (Uzawa, 1968; Epstein and Hynes, 1983; Epstein, 1987).

# Uzawa-Epstein Time Preference

- The following form of time-varying time preference is known as a version of Uzawa-Epstein habit formulation model.
  - Instantaneous discount rate is a function of consumption;
  - The welfare is the sum of instantaneous utilities, discounted by the cumulative discount rate up to time  $t$ .

$$W = \int_0^{\infty} u(C(t)) \cdot e^{-\Delta(t)} dt$$

$$\Delta(t) = \int_0^t r(C(s)) ds \quad \text{i.e.} \quad \frac{d\Delta}{dt} = r(C(t))$$

$$r(C(t)) > 0, \quad r'(C(t)) > 0, \quad r''(C(t)) \leq 0.$$

- In this model, the instantaneous discount rate at  $t$ ,  $d\Delta(t)/dt$  is increasing in consumption:
  - As the economic agent consumes more, its consumption attitude exhibits short-sighted and hedonistic.

# A study on exhaustible resource use

- Re-examined a classical topic of exhaustible resource use on the basis of recent development of models of time preference and discount factors.
- Analyzed the effects of endogenous time preference on dynamic properties of resource use in contrast to classical Hotelling's results.
  - Developed an analytical model that incorporates endogenous time preference into the decision framework of resource consumption.
  - The model structure:
    - cake-eating economy,
    - availability of a backstop technology, and
    - the Uzawa-Epstein formulation of time preference.

# Settings

- Consider a closed economy in which an exhaustible resource and a backstop technology are available.
- Suppose that there is no production sector.
  - i.e., the economic activity of the economy is restricted to consume the resource or utilize the available backstop technology.
- Assume that the population is constant.
- We introduce a representative agent who wishes to maximize the sum of discounted instantaneous utilities for the consumption of the exhaustible resource and equivalent resource consumption supplied by the backstop technology.



# The model

- We introduce the Uzawa-Epstein formulation of time preference.

We assume:

- instantaneous discount rate is a function of resource consumption;
- the sum of discounted instantaneous utilities is described as follows:

$$\int_0^T u(E(t)) \cdot e^{-\Delta(t)} dt + e^{-\Delta(T)} V$$

$$\Delta(t) = \int_0^t r(E(s)) ds$$

$$r(E(t)) > 0, \quad r'(E(t)) > 0, \quad r''(E(t)) \leq 0$$

$$V = \max_{\{\varepsilon(\tau)\}} \int_0^\infty (u(\varepsilon(\tau)) - P_B \varepsilon(\tau)) \cdot e^{-\Delta(\tau)} d\tau$$

$t$ : Time.

$E(t)$ : Exhaustible resource use at  $t$ .

$\Delta(t)$ : Cumulative discount rate at  $t$ .

$u(*)$ : Representative agent's instantaneous utility.

$r(*)$ : Instantaneous discount rate.

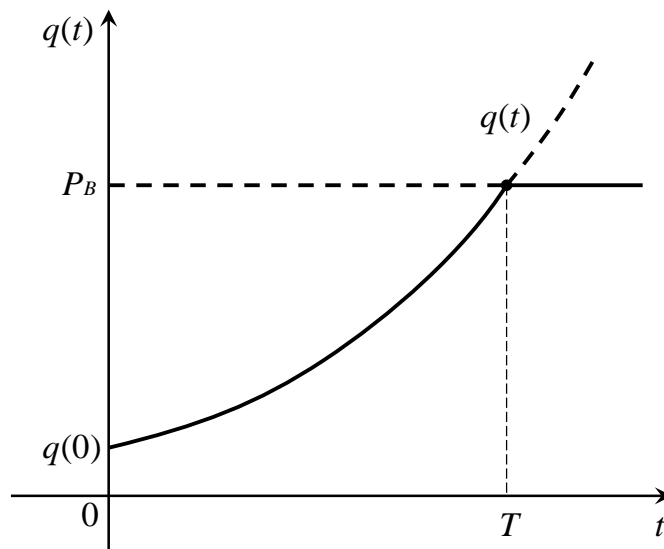
$P_B$ : The price of backstop technology.

$T$ : The time of switch.

- The last term  $V$ , represents the sum of discounted instantaneous utilities accruing from the use of backstop technology that starts operational at time  $T$  and that is used thereafter.

# Backstop Technology

- Backstop technology is the technology whose unlimited reserve is a substitute to the exhaustible resource stock.
  - Although the technology is physically available, it is too expensive at this moment.
- It may become economically available in future when the price of backstop technology becomes cheaper than that of the exhaustible resource.



$P_B$ : The price of backstop technology.  
 $q(t)$ : Scarcity rent of the exhaustible resource at time  $t$ .  
 $T$ : The time of switch.

# Optimal Use of Resources

- The maximization of the sum of discounted instantaneous utilities by the representative agent is formulated as the following optimization problem:

$$\max_{\{E(t)\}} \int_0^T u(E(t)) \cdot e^{-\Delta(t)} dt + e^{-\Delta(T)} V$$

$$\text{s.t.} \quad \frac{dS(t)}{dt} = -E(t) \quad \text{for} \quad 0 \leq t \leq T$$

$$\frac{d\Delta(t)}{dt} = r(E(t)) \quad \text{for} \quad 0 \leq t \leq T$$

$$S(0) = S_0, \text{ given.}$$

$t$ : Time.

$E(t)$ : Exhaustible resource use at  $t$ .

$S(t)$ : Stock of exhaustible resource at  $t$ .

$\Delta(t)$ : Cumulative discount rate at  $t$ .

$u(*)$ : Representative agent's instantaneous utility.

$r(*)$ : Instantaneous discount rate.

$P_B$ : The price of backstop technology

$$V = \max_{\{\varepsilon(\tau)\}} \int_0^\infty (u(\varepsilon(\tau)) - P_B \varepsilon(\tau)) \cdot e^{-\Delta(\tau)} d\tau \quad \text{s.t.} \quad \frac{d\Delta}{dt} = r(\varepsilon(t)) \quad \text{for} \quad T \leq t < \infty$$

# Key Parameters

- $\eta$  : the reciprocal of the intertemporal elasticity of substitution

$$u(E) = \frac{E^{1-\eta}}{1-\eta}, \quad \eta > 0, \quad \eta \neq 1$$

$$u'(E) > 0, \quad u''(E) < 0$$

$$-\frac{d \ln E}{d \ln u'} \equiv \frac{1}{\eta}$$

For large  $\eta$ , consumption is “inelastic.”  
For small  $\eta$ , consumption is “elastic.”

- Assumption 1

- The instantaneous discount rate at time  $t$  is linearly correlated to exhaustible resource consumption at time  $t$ .
- We call the coefficient,  $\beta$  “time preference coefficient.”

$$r = \beta E(t), \quad \beta > 0$$

$$r' = \beta, \quad r'' = 0$$

# Conditions for Optimality

- FONCs

$$u'(E(t)) - \beta \cdot \phi(t) = q(t)$$

$$\frac{dq(t)}{dt} = q(t) \cdot \beta E(t)$$

$$\frac{d\phi(t)}{dt} = \beta E(t) \cdot \phi(t) - u(E(t))$$

- Physical constraint

$$\int_0^T E(t) dt = S_0 - S(T)$$

- Terminal Conditions

$$q(T) = P_B$$

$$S(T) = 0$$

$u(t)$ : Representative agent's instantaneous utility.

$E(t)$ : Exhaustible resource use.

$S(t)$ : Stock of exhaustible resource.

$P_B$ : The price of backstop technology.

$q(t)$ : Current-valued scarcity rent of exhaustible resource.

$\phi(t)$ : Current-valued shadow prices for cumulated discount rate.

$T$ : The time of switch.

$\beta$ : Time preference coefficient.

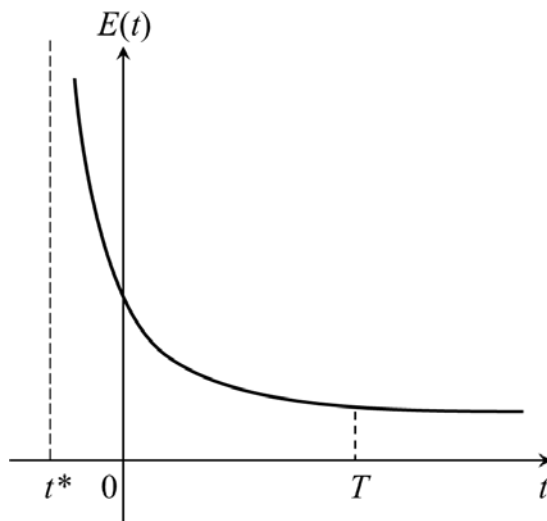
# Solutions

- Exhaustible resource consumption path

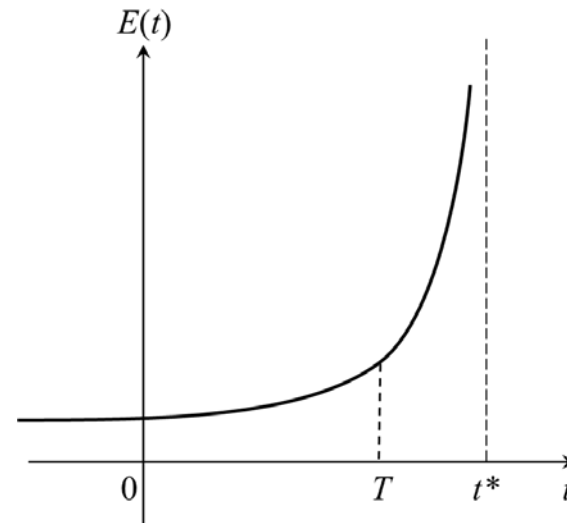
– must follow the following differential equation:  $\frac{dE(t)}{dt} = \frac{\beta}{1-\eta} E(t)^2$

Thus, 
$$E(t) = \frac{1}{P_B^{\frac{1}{\eta}} \cdot e^{\frac{\beta}{1-\eta} S_0 t} - \frac{\beta}{1-\eta} t}$$

- We obtain two cases, depending on the value of  $\eta$ .



$1 < \eta$



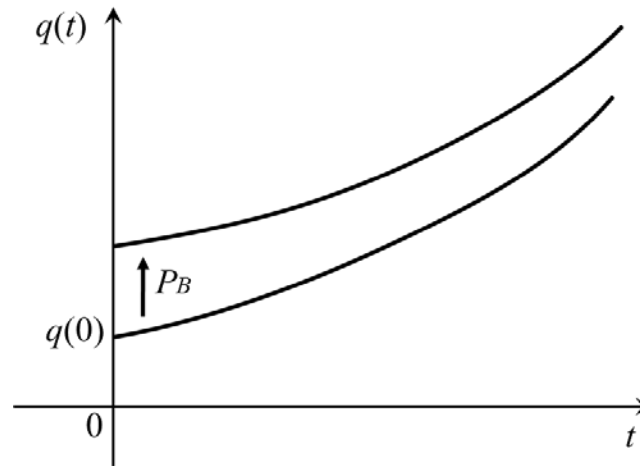
$0 < \eta < 1$

# Solutions, cont'd

- Scarcity rent path

$$q(t) = P_B^{\frac{1}{\eta}} \cdot E(t)^{1-\eta}$$

$$\frac{dq(0)}{dP_B} = e^{-\beta S_0} > 0$$



- It is increasing in  $t$  : As the exhaustible resource becomes scarce, the economic value becomes high. This fact is consistent to the idea of conventional Hotelling's rule.
- However, it should be underlined that the actual trajectory is completely different from that of Hotelling's rule in that it follows hyperbolic functions rather than exponential ones.

# Time to Switch

- The time to switch from exhaustible resource use to the use of backstop technology:

$$T = P_B^{\frac{1}{\eta}} \cdot \left( \frac{\beta}{1-\eta} \right)^{-1} \cdot \left( e^{\frac{\beta}{1-\eta} S_0} - 1 \right)$$

- Proposition 1.

*The increase in initial exhaustible resource stock,  $S_0$ , leads to the delay of the time to switch from exhaustible resource use to the use of backstop technology,  $T$ . The relation is determined as follows:*

$$\frac{dT}{dS_0} = P_B^{\frac{1}{\eta}} e^{\frac{\beta}{1-\eta} S_0} > 0$$

- Notice that the result is not directly affected by our introduction of the Uzawa-Epstein formulation. An abundance of exhaustible resource always leads the delay of the introduction of substitutes.

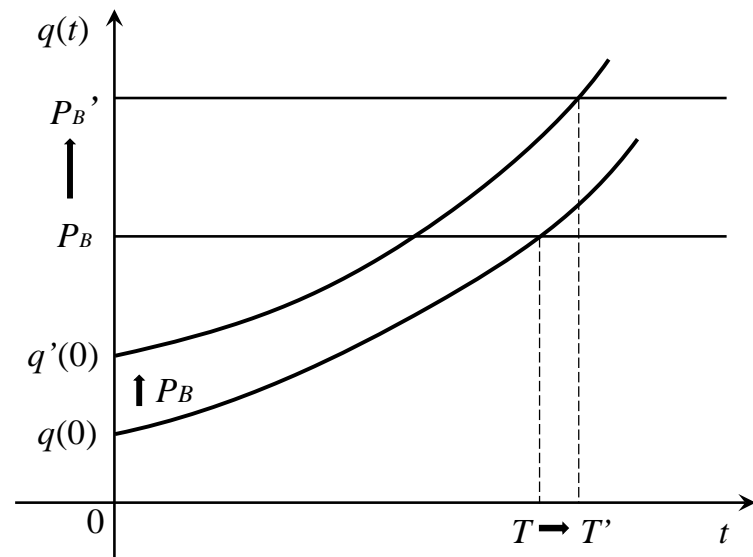


# Effects of Backstop Prices

- Proposition 2.

*The increase in the price of backstop technology,  $P_B$ , leads to the delay of the time to switch from exhaustible resource use to the use of backstop technology,  $T$ . The relation is determined as follows:*

$$\frac{dT/T}{dP_B/P_B} = \frac{1}{\eta} > 0$$



- The interpretation of the proposition is clear in that it is consistent with indications of conventional Hotelling's rule.

# Effects of Endogenous Discount Rates

- Proposition 3.

*The increase in the time preference coefficient,  $\beta$ , facilitates either earlier introduction or delay of backstop technology, depending on the value of the reciprocal of intertemporal elasticity of substitution,  $\eta$ . That is, the following relations hold true:*

(1) *In the case of  $1 < \eta$ ,*  $\frac{dT}{d\beta} < 0$  *(T comes earlier.)*  
*("inelastic" case)*

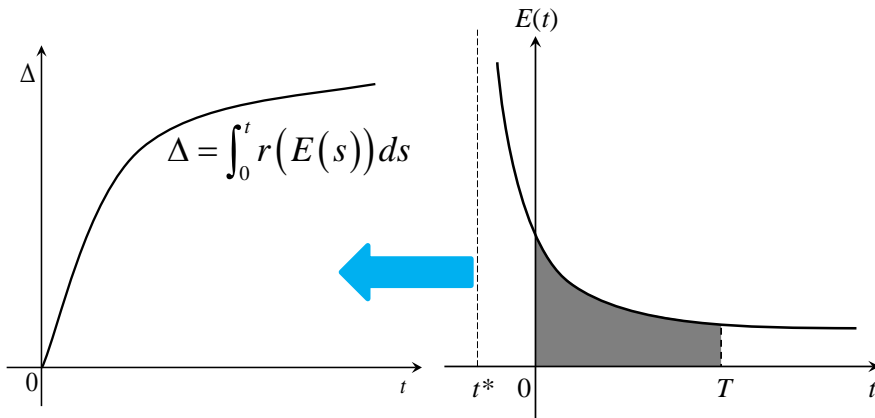
(2) *In the case of  $0 < \eta < 1$ ,*  $\frac{dT}{d\beta} > 0$  *(T moves forward.)*  
*("elastic" case)*

$$T = P_B^{\frac{1}{\eta}} \cdot \left( \frac{\beta}{1-\eta} \right)^{-1} \cdot \left( e^{\frac{\beta}{1-\eta} S_0} - 1 \right)$$

# Interpretation

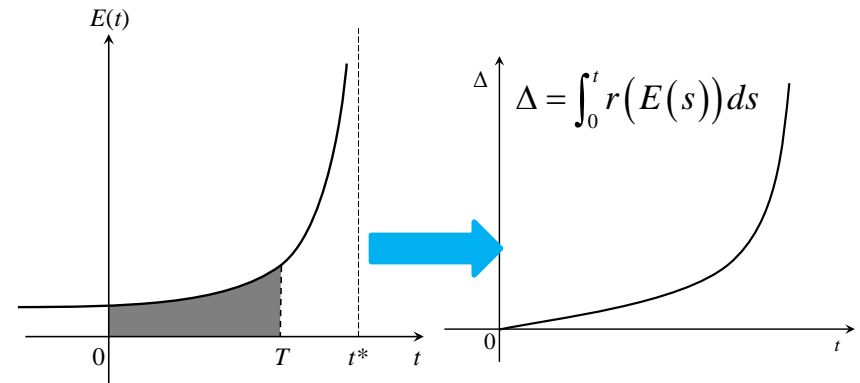
(1)  $1 < \eta$  (“inelastic” case):

Because exhaustible resource consumption is accumulated faster, short-sighted consumption habit is accelerated, which leads to earlier depletion of the resource.



(2)  $0 < \eta < 1$  (“elastic” case):

- Two effects may exist:
  - Because of small  $\eta$ , exhaustible resource consumption ( $E$ ) is elastic: It can easily be shifted along the time horizon.
  - Due to the shape of its trajectory, consumption habit is formed slowly, which leads to a less short-sighted consumption habit.
- As a result, depletion is delayed.



# Findings from the analytical model

- We investigated a classical model of the optimal use of exhaustible resource with the availability of backstop technology.
  - The new ingredient added to our model here is endogenous time preference that is determined by the history of resource consumption.
  - The dependence of time preference on consumption is interpreted as a form of habit formation, and is known as the Uzawa-Epstein formulation.
- We found that the reciprocal of the intertemporal elasticity of substitution ( $\eta$ ) in the instantaneous utilities determines the shapes of the trajectories of the exhaustible resource consumption.
- Proposition 3: The increase in the time preference coefficient,  $\beta$ , influences the time of switch, depending on the intertemporal elasticity of substitution.
  - The increase in  $\beta$  leads to an incentive for consuming the exhaustible resource faster (slower) and switching to the backstop technology earlier (later, resp.) as much as possible in the “inelastic” case (the “elastic” case, resp.).

# Conclusion

- While energy-environmental policy modeling and analysis benefits policy makers with policy recommendations, these are quite sensitive to the modeling frame and parameters, in particular time preference and discount factors.
- The concept of time preference and discounting is one of the most important inventions in economic theory, and thus has a long history of discussion on it, which still seems unresolved.
- Both policy makers and analysts should be aware of it.